Exercise 34

For the following exercises, solve the equations over the complex numbers.

$$x^2 + 6x + 25 = 0$$

Solution

The two terms with x, x^2 and $6x$, cannot be combined, so it's necessary to complete the square to solve for x. Recall the following algebraic identity.

$$(x + B)^2 = x^2 + 2xB + B^2$$

Notice that $2B = 6$, which means $B = 3$ and $B^2 = 9$. Add and subtract 9 from the left side and apply the identity.

$$(x^2 + 6x + 9) + 25 - 9 = 0$$

$$(x + 3)^2 + 16 = 0$$

Now that x appears in only one place, it can be solved for. Subtract 16 from both sides.

$$(x + 3)^2 = -16$$

Take the square root of both sides.

$$\begin{aligned}\sqrt{(x+3)^2} &= \sqrt{-16} \\ &= \sqrt{16(-1)} \\ &= \sqrt{16}\sqrt{-1} \\ &= 4i\end{aligned}$$

Since there's an even power under an even root, and the result is to an odd power, an absolute value sign is needed around $x + 3$.

$$|x + 3| = 4i$$

Remove the absolute value sign by placing $\pm$ on the right side.

$$x + 3 = \pm 4i$$

Subtract 3 from both sides.

$$x = -3 \pm 4i$$

Therefore, $x = \{-3 - 4i, -3 + 4i\}$.